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Quintessence, Flat Potential and String/M Theory Axion

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Abstract

A slow-rolling scalar field (Quintessence) has been proposed as the origin of accelerating universe at present. We discuss some features of quintessence, particularly those related with the *flatness* of its potential. We distinguish two types of quintessences, a metrical quintessence which couples to the particle physics sector *only* through the mixing with the spacetime metric and a non-metrical one without such property. It is stressed that these two types of quintessences have quite different features in regard to the fine tuning of parameters required for the flat potential, as well as different implications for the quintessence-mediated long range force. The fine tuning argument strongly suggests that non-metrical quintessence should correspond to the Goldstone boson of an almost exact nonlinear global symmetry whose explicit breaking by quantum gravity effects is highly suppressed. It is briefly discussed whether the radion field in higher dimensional theories can be a candidate for metrical quintessence. We finally discuss the possibility that string/M theory axion corresponds to a Goldstone-type quintessence, and argue that a certain combination of the heterotic M theory or Type I string theory axions can be a good candidate for quintessence if some conditions on the moduli dynamics are satisfied.

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I. INTRODUCTION

Recent measurements of the luminosity-red shift relation for Type Ia supernovae suggest that the Universe is accelerating and thus a large fraction of the energy density has negative pressure [1]. In fact, even before these supernovae data, there have been some indications that the Universe is dominated by an yet unknown form of smooth dark energy density [2]. The Big-Bang nucleosynthesis implies the baryon mass density $\Omega_B \sim 0.05$, while the measurements of cluster masses suggest the (clumped) dark matter mass density $\Omega_{DM} \sim 0.3$. (Here all mass densities are expressed in terms of their ratios to the critical mass density $\rho_c = 3M_P^2 H^2$.) On the other hand, the anisotropy of the cosmic microwave background shows the first peak of its angular power spectra at $l \sim 200$, implying that the total energy density $\Omega_{\rm TOT} \sim 1$ which is consistent with the prediction of inflation. This suggests the existence of a smooth (unclumped) dark energy density with $\Omega_X \sim 0.7$, and the supernovae data provide a strong support for such dark energy density with further information that it has negative pressure density $p_X \lesssim -0.6\Omega_X$.

If one accepts the accelerating universe as a reality, the utmost question is what is the nature of the dark energy with negative pressure. Of course, a nonvanishing cosmological constant, i.e. a non-evolving vacuum energy density, would be the simplest form of dark energy density with negative pressure. However there has been some prejudice that the correct, if exists any, solution to the cosmological constant problem [3] will lead to an exactly vanishing vacuum energy density. Recently quintessence in the form of slow-rolling scalar field has been proposed as an alternative form of dark energy density with negative pressure [4–6]. The true minimum of the quintessence potential is presumed to vanish. However the present value of Q is *displaced* from the true minimum, providing

$$V_Q \sim H_0^2 M_P^2 \sim (3 \times 10^{-3}\,\mathrm{eV})^4, \tag{1}$$

and a negative pressure with the equation of state:

$$\omega = \frac{p_Q}{\rho_Q} = \frac{\frac{1}{2}\dot{Q}^2 - V_Q}{\frac{1}{2}\dot{Q}^2 + V_Q} \lesssim -0.6, \tag{2}$$

where $M_P = 1/\sqrt{8\pi G_N} = 2.4 \times 10^{18}$ GeV denotes the reduced Planck scale and $H_0 \sim 10^{-32}$ eV is the Hubble expansion rate at present. In fact, when combined with the equation of motion for Q, the negative pressure value requires the following slow roll condition:

$$\left|\frac{\partial V_Q}{\partial Q}\right| \lesssim \left|\frac{V_Q}{M_P}\right|. \tag{3}$$

This condition indicates that if V_Q is well approximated by the terms with nontrivial Q-dependence, not by a Q-independent constant term, the typical range of Q is of the order of M_P or at least not far below M_P.

The basic idea of quintessence is to assume a scalar field whose potential energy V_Q is within the order of $H_0^2 M_P^2$ over the Planck scale range of Q. Then an utmost question is how such an extremely flat V_Q can arise from realistic particle physics models [7,8]. In section II, we discuss this issue together with some generic features of quintessence. We distinguish two types of quintessences, a Brans-Dicke type metrical quintessence which couples to the particle physics sector only through the mixing with the conformal factor of the spacetime metric and a non-metrical one without such property. These two types of quintessences have quite different features in regard to the fine tuning of parameters required for the flat potential, as well as different implications for the quintessence-mediated long range force. We argue that non-metrical quintessence should corrrespond to the Goldstone boson of an almost exact nonlinear global symmetry whose explicit breaking by quantum gravity effects is highly suppressed. Unless a Goldstone-type, non-metrical quintessence encounters a severe fine tuning problem which is much worse than the conventional cosmological constant problem. To make this point more clear, we briefly discuss the difficulties of a non-Goldstone type non-metrical quintessence having an inverse-power potential $V_Q = m^{4+\alpha}/Q^\alpha$. We also discuss whether the radion field in theories with (large) extra dimensions [9] can be a candidate for metrical quintessence [10]. In section III, we note that a certain combination of axions in heterotic M-theory [11] or Type I string theory [12,13] can be a plausible candidate for non-metrical quintessence [8] if its modulus partner is stabilized at a large value through the Kähler stabilization mechanism [14]. Although this quintessence axion could avoid the fine tunings of parameters associated with its flat potential, it still suffers from the fine tuning of cosmological initial conditions. In section IV, we discuss a late time inflation scenario based on the modular and CP invariance of the effective potential, which would solve this initial condition problem.

II. SOME GENERIC FEATURES OF QUINTESSENCE

The supernovae data indicate that the Universe has been accelerating since the redshift factor $z = \mathcal{O}(1)$, and thus over the time scale of $t_0 \sim 1/H_0 \sim 10^{10}$ yrs [1]. To have a negative pressure $p_Q = \dot{Q}^2/2 - V_Q$, one needs

$$\dot{Q} \lesssim \sqrt{V_Q} \sim H_0 M_P \tag{4}$$

over the accelerating period. In order for the above condition to be satisfied, one needs also

$$\ddot{Q} \lesssim H_0^2 M_P \tag{5}$$

over the same accelerating period. Note that we have ignored the coefficients of order one, and also $\dot{Q}$ and $\ddot{Q}$ can be regarded as appropriately averaged quantities. At any rate, when combined with the equation of motion

$$\ddot{Q} + 3H\dot{Q} + \frac{\partial V_Q}{\partial Q} = 0, \tag{6}$$

the Eqs.(4) and (5) lead to

$$\left|\frac{\partial V_Q}{\partial Q}\right| \lesssim \left|\frac{V_Q}{M_P}\right| \sim \frac{(3\times 10^{-3}\,\mathrm{eV})^4}{M_P}, \tag{7}$$

which has to be satisfied over the range of Q from the value at $z \sim 1$ to the present value.

To see the implications of the slow-roll condition (7), let us assume that V_Q is well approximated by

$$V_Q = V_0 + \frac{1}{2}m_Q^2 Q^2, \tag{8}$$

where V_0 is a Q-independent constant. If V_Q is dominated by the constant V_0 during the period of interest, Eq.(7) leads to

$$|Q/M_P| \lesssim (H_0/m_Q)^2 \sim (10^{-32}\,\mathrm{eV}/m_Q)^2 \tag{9}$$

and thus

$$|m_Q^2 Q^2/V_0| \lesssim H_0^2/m_Q^2 \sim (10^{-32}\,\mathrm{eV}/m_Q)^2. \tag{10}$$

This shows that if $m_Q \gg H_0 \sim 10^{-32}$ eV, we are in the limit of *non-evolving* cosmological constant in which Q is frozen at the minimum of V_Q. (Here m_Q^2 is assumed positive.)

However if the Q-dependent part $m_Q^2 Q^2/2$ dominates or at least non-negligible compared to the constant V_0, Eq.(7) leads to

$$|Q| \gtrsim M_P, \quad m_Q \lesssim 10^{-32}\,\mathrm{eV}, \tag{11}$$

and so V_Q *must be extremely flat over the Planck scale range of* Q. This is true even when $m_Q^2 Q^2/2$ is replaced by other forms of potential, e.g. the inverse power potential $m^{4+\alpha}/Q^\alpha$ or the exponential potential $m^4 e^{-Q/v}$. Thus if it has any feature distinguished from the non-evolving cosmological constant, the quintessence model involves an extremely flat potential, $V_Q \sim H_0^2 M_P^2 \sim (3\times 10^{-3}\,\mathrm{eV})^4$ over the Planck scale range of Q, and so an almost massless scalar boson with mass $m_Q \sim H_0 \sim 10^{-32}$ eV in rough order of magnitude estimate.

As is well known, there are rather strong observational limits on the couplings (and also their evolutions) between an almost massless scalar and normal matters [15]. For a discussion of these constraints and also of the fine tunings required for the flat potential, it is convenient to distinguish two types of quintessences: a metrical quintessence which couples to the particle physics sector only through the mixing with the spacetime metric, and a non-metrical one without such feature. Note that metrical quintessence is quite similar to the Brans-Dicke type scalar field in scalar-tensor gravity models. As we will see, these two types of quintessences have quite different features in the fine tuning of parameters for the flat potential, as well as its implications for the quintessence-mediated long range force.

Let us first consider non-metrical quintessence Q whose couplings to matter are *not* related with those of the spacetime metric. The interaction lagrangian can include

$$\mathcal{L}_{int} = \frac{Q}{M_P}(\xi_{F^2} F_{\mu\nu}F^{\mu\nu} + \xi_{G^2} G^a_{\mu\nu}G^{a\mu\nu} + \sum_q \xi_q m_q \bar{q}q), \tag{12}$$

where $F_{\mu\nu}$ and $G^a_{\mu\nu}$ denote the electromagnetic and gluon field strengths, respectively, and q stands for the light quarks with mass m_q. Note that ξ_I's represent the ratios of the quintessence couplings to the gravitational coupling. Since the operator combination which couples to Q does not match to the energy momentum tensor even in the non-relativistic limit, they violate the equivalence principle for non-relativistic laboratory bodies. More explicitly, the relevant operator combination can be written as

$$\xi_{G^2} G^{a\mu\nu}G^a_{\mu\nu} + \sum_q \xi_q m_q \bar{q}q = \frac{2g}{\beta(g)}\xi_{G^2} T^\mu_\mu + \sum_q [\xi_q - \frac{2g}{\beta(g)}\xi_{G^2}(1+\gamma_m)] m_q \bar{q}q, \tag{13}$$

where T^{μ}_{μ} is the trace of the QCD energy momentum tensor with the QCD beta function $\beta(g)$ and the mass anomalous dimension γ_m. Then the second term in the r.h.s. gives rise to a composition-dependent acceleration a_i of the i-th test body, yielding

$$\frac{a_i - a_j}{a_i + a_j} \approx \left(\frac{2g}{\beta(g)} \xi_{G^2} + \frac{\langle \sum_q \tilde{\xi}_q m_q \bar{q} q \rangle_s}{\langle T^{\mu}_{\mu} \rangle_s} \right) \left(\frac{\langle \sum_q \tilde{\xi}_q m_q \bar{q} q \rangle_i}{\langle T^{\mu}_{\mu} \rangle_i} - \frac{\langle \sum_q \tilde{\xi}_q m_q \bar{q} q \rangle_j}{\langle T^{\mu}_{\mu} \rangle_j} \right) \tag{14}$$

where $\tilde{\xi}_q = \xi_q - 2g\xi_{G^2}(1+\gamma_m)/\beta(g)$ and the subscript "s" denotes the source of the long range force. Under the assumption that there is no sizable cancellation between ξ_q and $2g\xi_{G^2}(1+\gamma_m)/\beta(g)$, which is the case for non-metrical quintessence, the current observational limit on non-universal acceleration [15]

$$|\Delta a/a| \lesssim 10^{-12} \tag{15}$$

leads to [16]

$$\xi_{G^2} \lesssim \mathcal{O}(10^{-5}), \quad \xi_q \lesssim \mathcal{O}(10^{-5}). \tag{16}$$

for the operator coefficients ξ_{G^2} and ξ_q renormalized at energy scale ~ 1 GeV.

If ξ_{F^2} is nonzero at low energies, say at $\mathcal{O}(1)$ eV, a time-varying Q leads to a time-varying fine structure constant which is constrained [15] as

$$\dot{\alpha}_{em}/\alpha_{em} \lesssim 10^{-16}\,\mathrm{yr}^{-1}. \tag{17}$$

Obviously this gives

$$\xi_{F^2}\dot{Q} \lesssim 3 \times 10^{-6} M_P H_0, \tag{18}$$

a rather strong observational limit on the combination $\xi_{F^2}\dot{Q}$.

The direct observational limits (16) already suggest that the non-derivative couplings between non-metrical Q and normal matter are highly suppressed, even compared to the gravitational strength. In fact, although indirect, much stronger limits on generic non-derivative couplings can be obtained from the fine tuning argument associated with the extremely flat potential. The quintessence couplings of Eq. (12) mean that the electromagnetic and QCD fine structure constants, and the light quark masses are all Q-dependent. Then the QCD and electroweak physics contribute to V_Q, yielding the following form of the quintessence potential in rough order of magnitude estimates

$$V_Q = \Lambda^4_{QCD} + m_q \Lambda^3_{QCD} + \frac{1}{16\pi^2} M_W^4 \ln(M_W^2/\mu^2) + ... \tag{19}$$

where the ellipsis stands for other types of contributions to V_Q. Here the QCD scale $\Lambda_{QCD} \propto e^{-2\pi/b\alpha_{QCD}(Q)}$ and the W-mass $M_W \propto \alpha_{em}(Q)$ are some functions of Q whose Q-dependence would be determined by the Q couplings to the gluon and electroweak field strengths.

Suppose we tuned the parameters of the theory to make $V_Q \sim (3 \times 10^{-3}\,\mathrm{eV})^4$ for *a particular value of* Q, say for $Q = Q_0$. Then for different value of Q, e.g. $Q = Q_0 + \Delta Q$, the three terms in V_Q of Eq.(19) would lead to a potential energy difference

$$
\begin{aligned}
\delta_1 V_Q &\sim \xi_{G^2}\Lambda_{QCD}^4 \frac{\Delta Q}{M_P} \sim 10^{43}\,\xi_{G^2} H_0^2 M_P^2 \frac{\Delta Q}{M_P}, \\
\delta_2 V_Q &\sim \xi_q m_q \Lambda_{QCD}^3 \frac{\Delta Q}{M_P} \sim 10^{42}\,\xi_q H_0^2 M_P^2 \frac{\Delta Q}{M_P}, \\
\delta_3 V_Q &\sim \ 10^{-2}\xi_{F^2} M_W^4 \frac{\Delta Q}{M_P} \sim 10^{52}\,\xi_{F^2} H_0^2 M_P^2 \frac{\Delta Q}{M_P}.
\end{aligned}
\tag{20}
$$

These estimates of the variation of V_Q show that even after the fine tunining for $V_Q \sim H_0^2 M_P^2$ at $Q = Q_0$, one still needs additional fine tunings in order to keep V_Q within the order of $H_0^2 M_P^2$ over the Planck scale range of Q. Generically, one needs *different* fine tunings for *different* values of Q, and then *the theory would suffer from the infinite numbers of fine tuning problems.*

There are essentially two ways to avoid this difficulty. The first one is to assume a non-linear continuous global symmetry ($U(1)_Q$) under which Q is shifted by a constant. Such $U(1)_Q$ would assure that all non-derivative couplings of Q to the particle physics sector are suppressed, so that the particle physics sector contribution to V_Q can be small enough for the wide range of Q. The second possibility is that although the non-derivative couplings between Q and normal matter are sizable, they are all precisely correlated to each other in such a way that their contributions to V_Q are cancelled *independently of* the values of Q. It is likely that the only sensible model in this direction is the metrical quintessence whose couplings to matter are induced *only* through the mixing with the spacetime metric. This leads us to conclude that, in order to avoid the infinite numbers of fine tuning problems, non-metrical quintessence should correspond to the Goldstone boson of a nonlinear global symmetry under which

$$
U_Q(1): \quad Q \to Q + \text{constant}. \tag{21}
$$

Still the potential difficulty for Goldstone-type quintessence is that it requires a nonzero but extremely tiny breaking of $U(1)_Q$. Note that generically a global symmetry can not be arbitrarily good, particularly when quantum gravity effects are included. So the questions are (1) how $U(1)_Q$ could avoid a potentially large breaking from quantum gravity effects and (2) what is the origin of the tiny $U(1)_Q$ breaking which would provide small but nonvanishing $V_Q \sim H_0^2 M_P^2$. To quantify this issue, let us first estimate the allowed size of $U(1)_Q$ breaking. Again assuming that $U(1)_Q$ breaking couplings of Eq.(12) are not correlated to each other, the variations of V_Q in Eq.(20) suggest that

$$
\xi_{F^2} \lesssim 10^{-52}, \quad \xi_{G^2} \lesssim 10^{-43}, \quad \xi_q \lesssim 10^{-42}, \tag{22}
$$

in order for V_Q remain to be within the order of $H_0^2 M_P^2$ over the Planck scale range of Q. This already shows that $U(1)_Q$-breaking couplings should be extremely suppressed, even compared to the gravitational coupling strength.

In fact, the variation (20) of V_Q includes only the contributions from the particle physics degrees of freedom with momenta $p \lesssim M_W$. If one includes the contributions from higher momentum modes, the typical variation of V_Q takes much larger value. As an illustrative example, let us consider generic supergravity models with cutoff scale $\Lambda \sim M_P$. The Kähler potential K, superpotential W, and gauge kinetic functions f_a can be expanded in powers of the visible sector matter fields Φ^i:

$$
\begin{aligned}
&K = K_0(Z, Z^*) + Z_{ij}(Z, Z^*)\Phi^i\Phi^{*j} + H_{ij}(Z, Z^*)\Phi^i\Phi^j + X_{ijk}(Z, Z^*)\Phi^i\Phi^j\Phi^{*k} + ..., \\
&W = W_0(Z) + Y_{ijk}(Z)\Phi^i\Phi^j\Phi^k + \Gamma_{ijkl}(Z)\Phi^i\Phi^j\Phi^k\Phi^l + ..., \\
&f_a = f_{a0}(Z) + f_{aij}(Z)\Phi^i\Phi^j + ...,
\end{aligned} \tag{23}
$$

where Z denotes the chiral multiplet including the quintessence degree of freedom Q. Any Q-dependence of the coefficient functions in the expansion gives rise to non-derivative couplings between Q and Φ^i, so reflects the explicit breaking of $U(1)_Q$. The typical sizes of the variation of V_Q induced by each of the Q-dependent coefficient functions are estimated in [8], taking into account various types of quantum corrections. Again if any of the non-derivative couplings is sizable, we can not keep V_Q small enough over the Planck scale range of Q by tuning finite number of parameters. This consideration leads to the following strong limits on the $U(1)_Q$ breaking couplings (in the unit with $M_P = 1$):

$$
\begin{aligned}
&\frac{\partial}{\partial Q}\ln(K_0) \lesssim 10^{-86}, \quad \frac{\partial}{\partial Q}\ln(W_0) \lesssim 10^{-86}, \\
&\frac{\partial}{\partial Q}Z_{ij} \lesssim 10^{-81}, \quad \frac{\partial}{\partial Q}\left|\frac{\partial H_{ij}}{\partial Z^*}\right|^2 \lesssim 10^{-74}, \\
&\frac{\partial}{\partial Q}\left|X_{ijk}\right|^2 \lesssim 10^{-74}, \quad \frac{\partial}{\partial Q}\left|Y_{ijk}\right|^2 \lesssim 10^{-79}, \\
&\frac{\partial}{\partial Q}\left|\Gamma_{ijkl}\right|^2 \lesssim 10^{-72}, \quad \frac{\partial}{\partial Q}\left|\frac{\partial f_{aij}}{\partial Z}\right|^2 \lesssim 10^{-67}, \\
&\frac{\partial}{\partial Q}\mathrm{Re}(f_{a0}) \lesssim 10^{-81}, \quad \frac{\partial}{\partial Q}\mathrm{Im}(f_{QCD}) \lesssim 10^{-42},
\end{aligned} \tag{24}
$$

where we have assumed an weak scale gravitino mass and the last bound is on the coupling of Q to the QCD anomaly $G^{a\mu\nu}\tilde{G}^a_{\mu\nu}$. In fact, even the flat potential argument does not give a meaningful limit on the coupling to the electroweak anomaly. So it is in principle possible that $\xi_{F\tilde{F}}$ is sizable for the coupling

$$
\xi_{F\tilde{F}}\frac{Q}{M_P}F^{\mu\nu}\tilde{F}_{\mu\nu}. \tag{25}
$$

Unless $|\xi_{F\tilde{F}}| \ll 10^{-2}$, such coupling may lead to an observable rotation of the polarization axis of the radiation from remote sources [16,17]. However in view of the gauge coupling unification, it is somewhat unlikely that Q has a sizable coupling to $F\tilde{F}$, while its coupling to $G\tilde{G}$ is so small as given by the last bound of Eq.(24).

The above limits on the $U(1)_Q$ breaking couplings suggest that $U(1)_Q$ must be an almost exact global symmetry. It is highly nontrivial to have such an almost exact global symmetry in any realistic models for particle physics once the effects of Planck scale physics are included through arbitrary higher dimensional operators in the effective action. In the next section, we will explore the possibility of such global symmetry in the framework of string/M theory and point out that a certain combination of the heterotic M theory or Type I string theory axions can be a plausible candidate for quintessence if some conditions on the moduli dynamics are satisfied. Here we stress that $U(1)_Q$ has nothing to do with a small V_Q at its minimum. It just ensures that V_Q is small over the Planck scale range of Q once it were small at its minimum. Without $U(1)_Q$, non-metrical quintessence model would suffer from infinite numbers of fine tuning problems to keep V_Q to be small over the wide range of Q. At any rate, the limits (22) which are based on the fine tuning argument suggest that it is hopeless to see the long range force mediated by non-metrical quintessence.

Let us now discuss the Brans-Dicke type metrical quintessence which couples to matter only through the mixing with the conformal factor of the spacetime metric. We first consider the observational limits on its couplings. The dynamics of metrical quintessence would be described by the following effective action:

$$S_{eff} = \int d^4x \left[\sqrt{-g}\left(-\frac{1}{2}M_P^2\mathcal{R}(g) + \frac{1}{2}g^{\mu\nu}\partial_\mu Q\partial_\nu Q - V_0(Q)\right) + \sqrt{-\tilde{g}}\,\mathcal{L}_m(\tilde{g}_{\mu\nu},\Phi)\right] \tag{26}$$

where $\mathcal{R}$ is the Ricci scalar density and $\mathcal{L}_m$ denotes the lagrangian density of generic matter fields Φ (including the conventional particle physics sector) under the background spacetime metric given by

$$\tilde{g}_{\mu\nu} = A^2(Q/M_P)g_{\mu\nu}. \tag{27}$$

Then the couplings between Q and Φ are given by

$$\mathcal{L}_{int} = Q\frac{\partial}{\partial Q}\left(\sqrt{-\tilde{g}}\mathcal{L}_m\right)_{Q=Q_0} = \xi\,\frac{Q}{M_P}T^\mu_\mu \tag{28}$$

where

$$\xi = M_P\left(\frac{\partial \ln A^2}{\partial Q}\right)_{Q=Q_0} \tag{29}$$

and

$$T^\mu_\mu = \frac{\beta(g)}{2g}G^{a\mu\nu}G^a_{\mu\nu} + \frac{\beta(e)}{2e}F^{\mu\nu}F_{\mu\nu} + \sum_i(1+\gamma_m)m_i\bar{\psi}_i\psi_i + ... \tag{30}$$

denotes the trace of the energy momentum tensor of the matter fields $\Phi = G^a_{\mu\nu}, F_{\mu\nu}, \psi_i$. Since it respects the equivalence principle in non-relativistic limit, the above quintessence couplings lead to a deviation from the Einstein's general relativity only through the relativistic corrections. Currently performed gravitational experiments in the solar system provide upper bounds on such deviation [15], implying

$$\xi \lesssim 4.5\times 10^{-2}. \tag{31}$$

One might think that the strong observational limit (17) on the time-varying fine structure constant can give a constraint on the evolution of metrical quintessence. However existing limits are derived from the electromagnetic processes with momentum transfer much smaller than the electron mass. For such low momentum transfer, we have $\beta(e) = 0$ and so a time-varying metrical quintessence does *not* lead to a time-varying electromagnetic fine structure constant.

Let us now turn to the flatness of the metrical quintessence potential. Since the couplings between Q and the particle physics sector are determined by the mixing with the spacetime metric, the whole contribution to V_Q from the particle physics sector is simply given by

$$\Delta S_{eff} \equiv \int d^4x\sqrt{-g}\,\Delta V_Q = \int d^4x\sqrt{-\tilde{g}}\Lambda_m^4 = \int d^4\sqrt{-g}A^4(Q/M_P)\Lambda_m^4, \tag{32}$$

where Λ_m^4 is a *Q-independent* energy density which is a generic function of all particle physics parameters in $\mathcal{L}_m$. Note that the general covariance w.r.t $\tilde{g}_{\mu\nu}$ guarantees the above form of ΔV_Q which is obtained after integrating out all particle physics sector fields. The total quintessence potential energy is then given by

$$V_Q = V_0 + \Delta V_Q = V_0(Q/M_P) + A^4(Q/M_P)\Lambda_m^4. \tag{33}$$

An interesting feature of the metrical quintessence potential is that once the particle physics parameters are adjusted to yield $\Lambda_m^4 \lesssim H_0^2 M_P^2$, the entire contribution to V_Q from the particle physics sector remains to be within the order of $H_0^2 M_P^2$ over the Planck scale range of Q. Of course one still needs to adjust the theory to keep V_0 to be small enough. The typical mass scales of the physics responsible for V_0 can be much lower than those of the conventional particle physics and/or the physics generating V_0 may enjoy extra symmetry which would assure that V_0 is small enough. In this case, one may be able to make the total $V_Q = V_0 + \Delta V_Q$ flat enough without a severe fine tuning problem other than the minimal fine tuning for small Λ_m^4. Note that if the (non-derivative) coupling between metrical quintessence and normal matter takes a value not far below the current observational limit (31), e.g. $\xi = \mathcal{O}(10^{-2})$, it can lead to observable effects in future gravity experiments.

An interesting candidate for metrical quintessence would be the radion field in theories with large extra dimensions [9,10]. However at least in the simplest framework, the radion has too strong coupling with matter to be a quintessence. For example, if one considers a higher dimensional theory with n large flat dimensions with common radius, the canonical radion field Q in the Einstein frame of the four dimensional metric $g_{\mu\nu}$ appears in the $(4+n)$-dimensional metric:

$$ds^2_{(4+n)} = g^{(4+n)}_{MN}dx^M dx^N = e^{\xi Q/M_P}g_{\mu\nu}dx^\mu dx^\nu + e^{-2\xi Q/nM_P}dy^i dy^i \tag{34}$$

where

$$\xi = \sqrt{2n/(n+2)}. \tag{35}$$

If the whole particle physics sector lives on a three brane, the radion couples to the particle physics sector only through the induced metric on the brane:

$$g^{(4+n)}_{\mu\nu} = e^{\xi Q/M_P}g_{\mu\nu}. \tag{36}$$

Obvioulsy, the value of ξ in this framework does not satisfy the observational limit (31), and thus the radion Q can not be a quintessence. (Of course, once one makes the radion massive by suitable stabilization mechanism [18], the limit (31) can be avoided.) Although a Brans-Dicke type quintessence can not arise from the simplest version, it may be possible to arrange the higher dimensional theory and the compactification scheme to yield an almost massless radion field with $\xi \lesssim \mathcal{O}(10^{-2})$.

Let us close this section with a brief discussion of the difficulties of specific non-metrical quintessence which is *not* Goldstone type, but has an inverse power law potential: $V_Q \sim m^{4+\alpha}/Q^\alpha$. Perhaps the most attractive feature of this model would be the tracker behavior that $\rho_Q \sim \rho_{matter}$ at present for wide range of initial conditions of Q and $\dot{Q}$. This feature allows us to avoid the cosmic coincidence puzzle asking why now ρ_Q is comparable to ρ_{matter} although their cosmological evolutions are generically different [5,6].

It has been pointed out [20] that the inverse power potential can be generated by non-perturbative dynamics in the underlying supersymmetric theory [21]. The model includes a hidden supersymmetric QCD sector with $SU(N_c)$ gauge group and N_f flavors of hidden quarks and antiquarks $\Phi \equiv (\Phi_i, \Phi_i^c)$. It is then assumed that the tree level superpotential is *independent* of Φ. The Φ-dependent part of the exact superpotential is then given by

$$W_{dyn} = \left(\frac{\Lambda^{3N_c - N_f}}{\det(\Phi_i \Phi_j^c)} \right)^{1/(N_c - N_f)}, \tag{37}$$

where Λ denotes the dynamical scale of the hidden $SU(N_c)$ gauge group. If W_{dyn} is a dominant term in the full superpotential, the effective potential of the $SU(N_f)$ invariant D-flat direction $\Phi_i = \Phi_i^c \equiv Q$ is given by

$$V_Q = \mathcal{F} \Lambda^{4+\alpha} / Q^\alpha, \tag{38}$$

where $\alpha = 2(N_c + N_f)/(N_c - N_f)$ and $\mathcal{F}$ is a function of Q/M_P which is essentially of order unity. The explicit form of $\mathcal{F}$ depends on the Kähler potential of Φ. For $\Lambda \sim M_P (H_0/M_P)^{2/(4+\alpha)} \sim 10^{-120/(4+\alpha)} M_P$, this potential is within the range of $H_0^2 M_P^2$ over the Planck scale range of Q, so can be a candidate for quintessence potential.

However the problem is that $W_{dyn} \sim H_0 M_P^2 \ll m_{3/2} M_P^2$ for $Q \sim M_P$, so it can *not* be a dominant term in the superpotential in any realsitic model of supersymmetry breaking. For $Q \sim M_P$, supergravity effects become important, and so we have to consider V_Q in the supergravity framework in which the scalar potential is schematically given by

$$V_{sugra} = |F|^2 + D^2 - \frac{3|W|^2}{M_P^2}, \tag{39}$$

where F and D denote the auxiliary F and D components of supersymmetry breaking fields, and W is the total superpotential. For realistic phenomenology, we need $|F| \gtrsim M_W^2$ (and/or $D \gtrsim M_W^2$) for the weak scale mass M_W. Obviously we then need $W \sim M_P |F| \sim m_{3/2} M_P^2$ in order to have the total vacuum energy density much smaller than M_W^4. Since W_{dyn} of Eq.(37) is just of the order of $H_0 M_P^2$, the total superpotential must include an additional term $W_0 \sim m_{3/2} M_P^2$. Once W_0 is introduced, which is a necessity, then the flatness of V_Q is totally lost.

To see this explicitly, let us consider the case with

$$\begin{aligned} W &= W_0 + W_{dyn} + \epsilon_1 \frac{(\Phi_i \Phi_i^c)^n}{M_P^{2n-3}} + ..., \\ K &= \Phi_i \Phi_i^* + \Phi_i^c \Phi^{c*} + \epsilon_2 \frac{(\Phi_i \Phi_i^*)^k}{M_P^{2k-2}} + ..., \end{aligned} \tag{40}$$

where n and k are some integers which can be arbitrarily large, and the ellipses stand for other possible terms. We then have (again schematically)

$$V_Q = \frac{\Lambda^{4+\alpha}}{Q^\alpha} + m_{3/2}^2 Q^2 + \epsilon_2 m_{3/2}^2 \frac{Q^{2k}}{M_P^{2k-2}} + \epsilon_1 m_{3/2} \frac{Q^{2n}}{M_P^{2n-3}} + \epsilon_1^2 \frac{Q^{4n-2}}{M_P^{4n-6}} + ..., \tag{41}$$

Obviously, for any realistic value of $m_{3/2}$, even after tuning the theory to have $V_Q \lesssim H_0^2 M_P^2$ at its minimum, one can not keep this V_Q within the range of $H_0^2 M_P^2$ over the Planck scale range of Q.

III. HETEROTIC M OR TYPE I STRING AXION AS QUINTESSENCE

In the previous section, we argued that non-metrical quintessence should corrrespond to the Goldstone boson of an almost exact nonlinear global symmetry in order to avoid infinite number of fine tuning problems associated with the flat potential. It is also noted that explicit breaking of this global symmetry by quantum gravity effects must be extremely tiny. In this section, we discuss the possibility that string and/or M theory axions correspond to such Goldstone-type quintessence. There are three different classes of string/M theory vacua which are particularly interesting in phenomenological sense: perturbative heterotic string vacua [13], heterotic M theory vacua [11], and Type I string vacua with D-brane configurations [12]. It is well known that axions in perturbative heterotic string vacua receives a large potential energy associated with the string world sheet instanton effects [19], so here we concentrate on heterotic M theory and Type I string vacua. We will see that certain combination of the heterotic M theory or Type I string theory axions can be a plausible candidate for quintessence if some conditions on the moduli dynamics are satisfied.

Let us first consider the heterotic M-theory on a 11-dimensional manifold with boundary which is invariant under the Z_2-parity [11]:

$$C \to -C, \quad x^{11} \to -x^{11}, \tag{42}$$

where $C = C_{ABC} dx^A dx^B dx^C$ is the 3-form field in the 11-dimensional supergravity. When compactified to 4-dimensions, axions arise as the massless modes of $C_{\mu\nu 11}$ and C_{mn11}:

$$\epsilon^{\mu\nu\rho\sigma} \partial_{[\nu} C_{\rho\sigma]11} = \partial^\mu \eta_S, \quad C_{mn11} = \sum_i \eta_i(x^\mu) \omega^i_{mn}, \tag{43}$$

where ω^i ($i = 1$ to $h_{1,1}$) form the basis of the integer $(1,1)$ cohomology of the internal 6-manifold and μ, ν are tangent to the noncompact 4-dimensional spacetime. In 4-dimensional effective supergravity, these axions appear as the pseudo-scalar components of chiral multiplets:

$$\begin{aligned} S &= (4\pi)^{-2/3} \kappa^{-4/3} V + i\eta_S, \\ T_i &= (4\pi)^{-1/3} \kappa^{-2/3} \int_{\mathcal{C}_i} \omega \wedge dx^{11} + i\eta_i, \end{aligned} \tag{44}$$

where κ^2 denotes the 11-dimensional gravitational coupling, V is the volume of the internal 6-manifold with the Kähler two form ω, and the integral is over the 11-th segment and also over the 2-cycle $\mathcal{C}_i$ dual to ω^i. Here the axion components are normalized by the discrete Peccei-Quinn (PQ) symmetries:

$$\text{Im}(S) \to \text{Im}(S) + 1, \quad \text{Im}(T_i) \to \text{Im}(T_i) + 1, \tag{45}$$

which are the parts of discrete modular symmetries.

Holomorphy and the discrete PQ symmetries imply that in the large $\mathrm{Re}(S)$ and $\mathrm{Re}(T_i)$ limits the gauge kinetic functions can be written as

$$4\pi f_a = k_a S + \sum_i l_{ai} T_i + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T_i}), \tag{46}$$

where k_a and l_{ai} are model-dependent *quantized* real constants and the exponentially suppressed terms are possibly due to the membrane or 5-brane instantons. For a wide class of compactifed heterotic M-theory, we have [22,23]

$$\begin{aligned} 4\pi f_{E_8} &= S + \sum_i l_i T_i + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T_i}), \\ 4\pi f_{E_8'} &= S - \sum_i l_i T_i + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T_i}), \end{aligned} \tag{47}$$

where $l_i T_i$ corresponds to the one-loop threshold correction in perturbative heterotic string terminology with the quantized coefficients l_i determined by the instanton numbers on the hidden wall and also the orbifold twists.

Let Q denote a linear combination of $\mathrm{Im}(T_i)$ *orthogonal* to the combination $\sum_i l_i \mathrm{Im}(T_i)$ and define the associated nonlinear global symmetry:

$$U(1)_Q : \quad Q \to Q + \text{constant}. \tag{48}$$

Note that such Q exists always as long as $h_{1,1} > 1$. In this regard, models of particular interest are the recently discovered threshold-free models with $l_i = 0$ [23] for which any of $\mathrm{Im}(T_i)$ can be identified as Q. At any rate, Q can be a candidate for quintessence if $U(1)_Q$ breaking couplings in the effective supergravity are all suppressed as in Eq.(24).

It is easy to see that for Q defined as above

$$\frac{\partial f_a}{\partial Q} = \mathcal{O}(e^{-2\pi T}), \tag{49}$$

over the entire range of Q. Here we use the unit with $M_P = 1$ and the internal 6-manifold is assumed to be isotropic, so $\mathrm{Re}(T_i) \approx \mathrm{Re}(T)$ for all T_i. Holomorphy and discrete PQ symmetries imply also

$$\frac{\partial Y_{ijk}}{\partial Q} = \mathcal{O}(e^{-2\pi T}) \tag{50}$$

for the Yukawa couplings Y_{ijk} in the superpotential. Similar estimate applies also to the non-renormalizable holomorphic couplings in the superpotential. A non-perturbative superpotential W_0 may be induced by these holomorphic gauge and superpotential couplings, e.g. by gaugino condensation. The $U(1)_Q$ breaking in W_0 is estimated to be

$$\frac{\partial W_0}{\partial Q} = \frac{\partial W_0}{\partial f_a}\frac{\partial f_a}{\partial Q} + \frac{\partial W_0}{\partial Y_{ijk}}\frac{\partial Y_{ijk}}{\partial Q} + ... = \mathcal{O}(e^{-2\pi T} W_0). \tag{51}$$

We then have

$$\frac{\partial W}{\partial Q} = \mathcal{O}(e^{-2\pi T} W) \tag{52}$$

for the full superpotential W.

The above estimates of $U(1)_Q$ breaking couplings in f_a and W are made by simple macroscopic argument based on supersymmetry and discrete PQ symmetries. However one can easily identify its microscopic origin by noting that $2\pi \mathrm{Re}(T_i) = (4\pi)^{-1/3}\kappa^{-2/3}\int_{\mathcal{C}_i}\omega \wedge dx^{11}$ corresponds to the Euclidean action of the membrane instanton wrapping $\mathcal{C}_i$ and stretched along the 11-th segment [22]. When extrapolated to the perturbative heterotic string vacua, such membrane instanton corresponds to the heterotic string worldsheet instanton wrapping the same 2-cycle [19]. Explicit computations then show that the Kähler potential and/or the gauge kinetic functions are indeed corrected by worldsheet instantons, yielding $\delta K = \mathcal{O}(e^{-2\pi T})$ and $\delta f_a = \mathcal{O}(e^{-2\pi T})$ [24]. These corrections can be smoothly extrapolated back to the heterotic M-theory vacua [26] and identified as the corrections induced by stretched membrane instanton. If a nonperturbative superpotential W_0 is generated by gaugino condensation, $\delta f_a = \mathcal{O}(e^{-2\pi T})$ leaves its trace in $\delta W_0 = \mathcal{O}(e^{-2\pi T} W_0)$. We thus conclude that $U(1)_Q$ breaking terms of $\mathcal{O}(e^{-2\pi T})$ are indeed induced in K, W and f_a for generic heterotic M theory vacua.

The holomorphy and discrete PQ symmetries (45) ensure that $U(1)_Q$ breaking terms in f_a and W, whatever their microscopic origin is, are all suppressed by $e^{-2\pi T}$, Also the $U(1)_Q$ breaking terms in K induced by stretched membrane instanton are suppressed by $e^{-2\pi T}$, However there may be unsuppressed $U(1)_Q$ breaking term in non-holomorphic K, which would arise from yet unknown microscopic origin. Although not definite, it is unlikely to have such unsuppressed correction. $U(1)_Q$ is a linear combination of $U(1)_i : \delta T_i = ic_i$ ($c_i =$ real constant) which originate from the *local* transformation of the 11-dimensional three form field: $\delta C = \omega^i \wedge dx^{11}$ with $c_i = \int \delta C = \int \omega^i \wedge dx^{11}$. This indicates that upon ignoring the effects of boundary degrees of freedom, the zero momentum mode of $\mathrm{Im}(T_i)$ couples only to stretched membrane instantons wrapping the 2-cycle $\mathcal{C}_i$ and are stretched between the boundaries. Since we chose the combination to avoid the breaking by Yang-Mills instantons on the boundary, $U(1)_Q$ appears to be broken only by stretched membrane instantons whose effects are suppressed by $e^{-2\pi T}$. It is thus expected that

$$\frac{\partial K}{\partial Q} = \mathcal{O}(e^{-2\pi T}) \tag{53}$$

as in the case of f_a and W.

Obviouslly Eqs. (49), (52), and (53) show that $U(1)_Q$ becomes an almost exact global symmetry in the limit $\mathrm{Re}(T) \gg 1$. From the supergravity potential

$$V_{sugra} = e^K \left[K^{I\bar{J}} D_I W (D_J W)^* - 3|W|^2\right] \tag{54}$$

and also the standard order of magnitude relations $m_{3/2} \sim W/M_P^2 \sim D_I W/M_P$, one easily finds that the axion potential is given by

$$V_Q \sim e^{-2\pi \mathrm{Re}(T)} m_{3/2}^2 M_P^2 \cos[2\pi \mathrm{Im}(T)]. \tag{55}$$

This axion potential can be identified as the quintessence potential $V_Q \sim (3 \times 10^{-3}\mathrm{eV})^4$ if the modulus vacuum value is given by

$$\mathrm{Re}(T) \sim \frac{1}{2\pi}\ln(m_{3/2}^2 M_P^2/V_Q) \sim 32, \tag{56}$$

where $m_{3/2} \sim 10^2$ GeV are used for numerical estimate.

In the above discussion, gauge kinetic functions are assumed to be given by (47). In such cases, Q must be a linear combination of $\mathrm{Im}(T_i)$ to avoid the couplings to the QCD and/or the hidden gauge anomaly. Gauge kinetic functions can be generalized to the form of (46) with *non-universal* k_a, and then Q can include the $\mathrm{Im}(S)$ component. At any rate, as long as the quantized coefficients k_a and l_{ai} are all of order unity, the domain of moduli space allowing a quintessence axion in heterotic M theory has

$$\mathrm{Re}(S) = \mathcal{O}\left(\frac{1}{\alpha_{GUT}}\right), \quad \mathrm{Re}(T) = \mathcal{O}\left(\frac{1}{\alpha_{GUT}}\right). \tag{57}$$

On this domain, we also have

$$\begin{aligned} \frac{M_{GUT}}{M_P} &\sim [\mathrm{Re}(S)\mathrm{Re}(T)]^{-1/2} = \mathcal{O}(\alpha_{GUT}), \\ \frac{\kappa^{2/3}}{(\pi\rho)^3} &\sim \mathrm{Re}(S)/[\mathrm{Re}(T)]^3 = \mathcal{O}(\alpha_{GUT}^2), \end{aligned} \tag{58}$$

implying that this domain gives the unification scale M_{GUT} close to the phenomenologically favored value 3×10^{16} GeV, and also the length $\pi\rho$ of the 11-th segment about one order of magnitude bigger than the 11-dimensional Planck length $\kappa^{2/9}$.

Let us now turn to Type I string axions. The Type I axions (again normalized by the discrete PQ symmetries of (45)) correspond to the massless modes of the R-R two form fields $B_{\mu\nu}$ and B_{mn}:

$$\epsilon^{\mu\nu\rho\sigma}\partial_\nu B_{\rho\sigma} = \partial^\mu \eta_S, \quad B_{mn} = \sum_i \eta_i(x^\mu)\omega^i_{mn}, \tag{59}$$

and they form 4-dimensional chiral multiplets together with the string dilaton e^D and the internal space volume V:

$$\begin{aligned} S &= (2\pi)^{-6}\alpha'^{-3}e^{-D}V + i\eta_S, \\ T_i &= (2\pi)^{-2}\alpha'^{-1}e^{-D}\int_{\mathcal{C}_i}\omega + i\eta_i. \end{aligned} \tag{60}$$

Here we include $D9$ and $D5$ branes in the vaccum configuration, and consider the 4-dimensional gauge couplings α_9 and α_{5i} defined on $D9$ branes wrapping the internal 6-manifold and $D5$ branes wrapping the 2-cycles $\mathcal{C}_i$, respectively [12]. Again holomorphy and discrete PQ symmetries imply that the corresponding gauge kinetic functions can be written as (46). A simple leading order calculation gives $\alpha_9 = 1/\mathrm{Re}(S)$ and $\alpha_{5i} = 1/\mathrm{Re}(T_i)$, and so [12]

$$4\pi f_9 = S, \quad 4\pi f_{5i} = T_i. \tag{61}$$

However this leading order result can receive perturbative and/or non-perturbative corrections. Generic perturbative corrections can be expanded in powers of the string coupling

e^D, the string inverse tension α', and also the inverse tension $e^D\alpha'^k$ of D_{2k-1} branes [13]. Generically they scale as

$$e^{nD}\alpha'^m \propto [\mathrm{Re}(S)]^{\frac{n-m}{2}}[\mathrm{Re}(T)]^{\frac{m-3n}{2}}. \tag{62}$$

Combined with the leading order result (61) and also the general form of gauge kinetic function (46) dictated by holomorphy and discrete PQ symmetries, this scaling behavior implies that f_9 can receive a T_i-dependent correction at order α'^2, while there is no perturbative correction to f_{5i}, and so

$$\begin{aligned} 4\pi f_9 &= S + l_i T_i + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T_i}), \\ 4\pi f_{5i} &= T_i + \mathcal{O}(e^{-2\pi S}, e^{-2\pi T_i}). \end{aligned} \tag{63}$$

Similarly to the case of heterotic M-theory, the quintessence axion Q can arise as a linear combination of $\mathrm{Im}(S)$ and $\mathrm{Im}(T_i)$, however its explicit form depends on how the various gauge couplings are embedded in the model. Note that for the QCD and even stronger hidden sector gauge couplings, both $\mathrm{Re}(f_a)$ and $\mathrm{Im}(f_a)$ are required to be Q-independent as in (24), while for the weaker gauge interactions $\mathrm{Im}(f_a)$ are allowed to have a sizable Q-dependence. Here are some possibilities. If all of the standard model gauge couplings and the stronger hidden sector gauge couplings are embedded in α_{5i}, $\mathrm{Im}(S)$ can be a quintessence when $\mathrm{Re}(S) \sim 32$. If those gauge couplings are embedded in α_9, any of $\mathrm{Im}(T_i)$ can be a quintessence when $\mathrm{Re}(T_i) \sim 32$. If the vacuum does not include any $D5$ brane wrapping the particular 2-cycle, e.g. the i-th cycle, the corresponding axion $\mathrm{Im}(T_i)$ can be a quintessence independently of the embedding when $\mathrm{Re}(T_i) \sim 32$.

For the quintessence component Q defined as above, the Q-dependences of K, W and f_a are all suppressed by $e^{-2\pi Z}$ ($Z = S$ or T) as in the case of heterotic M-theory. So the quintessence axion potential is again given by (55) where now T is replaced by Z. Microscopic origin of this Type I axion potential can be easily identified also by noting that $2\pi\mathrm{Re}(S)$ corresponds to the Euclidean action of $D5$ brane instanton wrapping the internal 6-manifold and $2\pi\mathrm{Re}(T_i)$ is of $D1$ string instanton wrapping the 2-cycle $\mathcal{C}_i$. Quintessence axion in Type I string theory requires also that both $\mathrm{Re}(S)$ and $\mathrm{Re}(T)$ are of $\mathcal{O}(1/\alpha_{GUT})$. On this domain of moduli space, we have

$$\begin{aligned} &e^{2D} \sim \mathrm{Re}(S)/[\mathrm{Re}(T)]^3 = \mathcal{O}(\alpha_{GUT}^2), \\ &\alpha'/V^{1/3} \sim [\mathrm{Re}(T)/\mathrm{Re}(S)]^{1/2} = \mathcal{O}(1), \end{aligned} \tag{64}$$

and thus a rather weak string coupling and also a rather strong sigma model coupling.

So far, we have noted that a certain combination of axions in heterotic M theory or Type I string theory can be a candidate for quintessence if its modulus partner has a large vacuum value. For the axion component normalized as $\mathrm{Im}(Z) \equiv \mathrm{Im}(Z) + 1$, explicit breaking of the associated global symmetry ($U(1)_Q : \mathrm{Im}(Z) \to \mathrm{Im}(Z) + \text{constant}$) is suppressed by $e^{-2\pi Z}$. More concretely, the effective supergravity model is described by

$$\begin{aligned} K &= \tilde{K}(Z + Z^*) + \mathcal{O}(e^{-2\pi Z}), \\ f_a &= \tilde{f}_a + \mathcal{O}(e^{-2\pi Z}), \\ W &= [1 + \mathcal{O}(e^{-2\pi Z})]\tilde{W}, \end{aligned} \tag{65}$$

where f_a denote the gauge kinetic functions for the standard model gauge group and also the stronger hidden sector gauge group, and $\tilde{f}_a$ and $\tilde{W}$ are Z-*independent*. We then have

$$\mathcal{L}_Q = \frac{1}{2}(\partial_\mu Q)^2 - m^4[\cos(Q/v_Q) + 1], \tag{66}$$

where

$$m^4 \sim e^{-2\pi \mathrm{Re}(Z)} m_{3/2}^2 M_P^2 \sim (3 \times 10^{-3}\mathrm{eV})^4, \tag{67}$$

for $\mathrm{Re}(Z) \sim 32$, and the canonical quintessence axion Q and its decay constant v_Q are given by

$$Q = M_P \sqrt{2K''}\,\mathrm{Im}(Z), \quad v_Q = \frac{1}{2\pi} M_P \sqrt{2K''}, \tag{68}$$

where K'' denotes the Kähler metric of Z:

$$K'' = \frac{\partial^2 K}{\partial Z \partial Z^*}. \tag{69}$$

At leading order approximation in perturbation theory, we have $K = c\ln(Z + Z^*)$ for a constant c of order unity, and then $\mathrm{Re}(Z)$ can not be stabilized at the desired value. Obviously W is almost Z-independent for large $\mathrm{Re}(Z)$, so can not provide a stabilizing potential of $\mathrm{Re}(Z)$. Note that if W provides any sizable potential of $\mathrm{Re}(Z)$, it means also a sizable potential of $\mathrm{Im}(Z)$, which is not allowed for $\mathrm{Im}(Z)$ to be a quintessence. We thus need a rather strong $U(1)_Q$-preserving nonperturbative effects encoded in the Kähler potential, e.g. $K_{np} \sim (Z + Z^*)^k e^{-b\sqrt{Z+Z^*}}$, which would stabilize $\mathrm{Re}(Z)$ at $\mathrm{Re}(Z) \sim 32$ [14]. As we will discuss in the next section, such strong correction to K is required also to enlarge K'' for cosmological reasons.

IV. INITIAL CONDITION PROBLEM AND A LATE TIME INFLATION SOLUTION:

In order to suppress $U(1)_Q$ breaking quantum gravity effects, we needed a large vacuum value of $\mathrm{Re}(Z)$. A large value of $\mathrm{Re}(Z)$ then implies $K'' = \mathcal{O}([\mathrm{Re}(Z)]^{-2}) \ll 1$, and so $v_Q \ll M_P$. (See Eq. (68).) Since v_Q determines the coupling strength of Q, while M_P determines the gravitational coupling strength, Q responds to its potential energy more sensitively than the expanding universe does. As a result, generic initial values of Q and $\dot{Q}$ give a rapidly rolling Q and thus positive pressure at present. To avoid this, we need a fine tuning of initial condition. So the attempt to avoid the fine tunings of parameters for the flat potential leads to a new fine tuning problem. In this section, we propose a late time inflation scenario based on the modular and CP invariance which would solve this initial condition problem.

Let us first consider what kind of initial conditions we need to have. When applied for (66) and (68), the slow roll condition (7) leads to

$$
\begin{aligned}
&|2\pi \mathrm{Im}(Z)| \lesssim \sqrt{2K''}/2\pi = \mathcal{O}\left(\frac{1}{2\pi \mathrm{Re}(Z)}\right), \\
&|2\pi \mathrm{Im}(\dot{Z})| \lesssim 2\pi H_0/\sqrt{2K''} = \mathcal{O}\left(\frac{2\pi H_0}{\mathrm{Re}(Z)}\right),
\end{aligned} \tag{70}
$$

implying that $\mathrm{Im}(Z)$ should be at near the top of its potential. The main difficulty of this condition is that

$$
m_Q \equiv \left|\frac{\partial^2 V_Q}{\partial Q^2}\right|^{1/2}_{Q=0} = \frac{m^2}{v_Q} \approx \frac{\sqrt{3}\pi}{\sqrt{K''}} H_0 \gg H_0, \tag{71}
$$

and thus a small $\mathrm{Im}(Z)$ is unstable against the cosmlogical evolution during the period with an expansion rate $H \lesssim m_Q$. The resulting instability factor is given by $e^{\gamma m_Q/H_0}$ where γ is a constant of order unity whose precise value depends upon the initial conditions. A detailed numerical study of the cosmological evolution gives $\gamma \approx 0.5$ for wide range of relevant initial conditions [25]. Thus the slow-roll condition (70) requires eventually the following initial conditions

$$
\begin{aligned}
&|2\pi \mathrm{Im}(Z)|_{in} \lesssim e^{-\gamma m_Q/H_0} \sim e^{-3/\sqrt{K''}}, \\
&|2\pi \mathrm{Im}(\dot{Z})|_{in} \lesssim H e^{-\gamma m_Q/H_0} \sim H e^{-3/\sqrt{K''}},
\end{aligned} \tag{72}
$$

for the period with $H \gg m_Q$.

Eq. (72) shows that the degree of required fine tuning is quite sensitive to the value of the Kähler metric K''. At leading order approximation in string or M theory, we have [13] $K \approx -c\ln(Z+Z^*)$ where c is a constant of order unity, so $K'' = \mathcal{O}(10^{-3})$ for $\mathrm{Re}(Z) \sim 30$. As we will see, the degree of fine tuning for this value of K'' is too severe to be accommodated, so we need a mechanism to enlarge the value of K''. It is expected that K'' can be enlarged by $U(1)_Q$-preserving nonperturbative effects which would be responsible for stabilizing $\mathrm{Re}(Z)$ [14]. However it is hard to imagine that K'' becomes of order unity, which would allow to avoid the fine tuning of initial condition without any further mechanism. In the following, we discuss a late time inflation scenario which would resolve the fine tuning problem of initial conditions for a reasonably enlarged value of K''.

The gauge symmetries of string or M-theory include discrete modular group [13] under which Z and other generic moduli ϕ transform as

$$
\mathrm{Re}(Z) \to \frac{1}{\mathrm{Re}(Z)}, \quad \mathrm{Im}(Z) \to \mathrm{Im}(Z) + 1, \quad \phi \to \phi', \tag{73}
$$

and also CP [27] under which

$$
Z \to Z^*, \quad \phi \to \phi^*. \tag{74}
$$

Here we take the simplest form of the Z-duality ($Z = S$ or T), i.e. $\mathrm{Re}(Z) \to 1/\mathrm{Re}(Z)$ with the self-dual value $\mathrm{Re}(Z) = 1$, however our discussion is valid for other forms of the Z-duality transformation as long as the self-dual value is of order unity.

The modular and CP invariance ensure that the invariant points,

$$\mathrm{Re}(Z) = 1, \quad \mathrm{Im}(Z) = 0 \ \text{ or } \ \frac{1}{2}, \quad \phi = \phi^* \ \text{ or } \ \phi'^*, \tag{75}$$

correspond to the stationary points of the effective action [28]. It is then quite possible that the modular invariant point $Z = 1$ is a (local) minimum of the effective potential *during the inflationary period* if the inflaton is a modular invariant field. This (local) minimum may become unstable if the inflaton field takes the present value, making $\mathrm{Re}(Z)$ rolls toward the present minimum at $\mathrm{Re}(Z) \sim 32$ after inflation. At any rate, during the inflationary phase, $\mathrm{Re}(Z) \sim 1$ and then all the moduli masses including that of $\mathrm{Im}(Z)$ have the same order of magnitude:

$$m_{\mathrm{Re}(Z)} \sim m_{\mathrm{Im}(Z)} \sim m_\phi \sim H_{inf}, \tag{76}$$

where H_{inf} denotes the expansion rate during inflation. Note that the axion mass $m_{\mathrm{Im}(Z)}$ is unsuppressed for $\mathrm{Re}(Z) \sim 1$. Since $\mathrm{Re}(Z)$ was far away from the present value, it is expected that the inflationary potential is at least of $\mathcal{O}(m_{3/2}^2 M_P^2)$, and so H_{inf} is at least of $\mathcal{O}(m_{3/2})$. To avoid a too large quantum fluctuation during this inflation, we take the minimal value of H_{inf}, so

$$H_{inf} = \mathcal{O}(m_{3/2}). \tag{77}$$

About the location of the minimum in the axion direction, we have just two possibilities if CP is *not* spontaneously broken in the moduli sector. One of the two CP invariant points, $\mathrm{Im}(Z) = 0$ and $1/2$, is the minimum, while the other is the maximum. *Our key assumption is that* $\mathrm{Im}(Z) = 0$ *was the minimum for the inflationary modulus value* $\mathrm{Re}(Z) = 1$*, however it becomes the maximum for the present modulus value* $\mathrm{Re}(Z) \sim 32$. Note that the coefficient of the cosine potential of $\mathrm{Im}(Z)$ is a function of $\mathrm{Re}(Z)$, and so its sign can be changed when $\mathrm{Re}(Z)$ varies from the inflationary value to the present value.

Given the features of the moduli potential discussed above, during the inflationary period all moduli are settled down near at the modular and CP-invariant local minimum with $\mathrm{Re}(Z) \approx 1$, $\mathrm{Im}(Z) \approx 0$, and $\phi \approx \phi^*$. In particular, just after inflation, we have [29]

$$\begin{aligned} |2\pi \mathrm{Im}(Z)|_{in} &= \mathcal{O}(e^{-3N_e/2}) + \mathcal{O}(H_{inf}/M_P), \\ |2\pi \mathrm{Im}(\dot{Z})|_{in} &= \mathcal{O}(e^{-3N_e/2} H_{inf}) + \mathcal{O}(H_{inf}^2/M_P), \end{aligned} \tag{78}$$

where N_e denotes the number of efoldings. Here the exponential suppression is due to the classical evolution toward the minimum of the inflationary potential, while the second terms represent the quantum fluctuations.

After this late inflation, $\mathrm{Re}(Z)$ rolls toward the present minimum at $\mathrm{Re}(Z) \sim 32$. In this period, the coefficient of the axion potential changes its sign, and thus $\mathrm{Im}(Z) = 0$ which was the minimum at the inflationary phase becomes the maximum of the present axion potential. The small value of $\mathrm{Im}(Z)$, i.e. (78), which was set during the inflation becomes unstable if H starts to be smaller than m_Q. However if the inflationary values (78) satisfy the condition (72), the present value of the quintessence axion satisfies (70), and thus provides an accelerating universe at present. This requires a large number of efolding

$$N_e \gtrsim \frac{2\gamma m_Q}{3H_0} \approx \frac{2}{\sqrt{K''}}, \tag{79}$$

and also a small quantum fluctuation

$$\delta Q \sim \frac{H_{inf}}{2\pi} \lesssim M_P e^{-3/\sqrt{K''}}. \tag{80}$$

However the number of efolding can not be arbitrarily large. For a relatively late inflation with $H_{inf} \sim m_{3/2}$, it is extremely diffcult to generate the observed density fluctuation $\delta\rho/\rho \sim 10^{-5}$. It is thus reasonable to assume that density fluctuations were created before the late inflation. In this case, the late inflation is required not to destroy the pre-exisiting density fluctuations. This consideration gives an upper bound on N_e [29], yielding

$$N_e \lesssim 25 \sim 30 \tag{81}$$

for a weak scale $m_{3/2}$ and the reheat temperature range $T_r = 10^5 \sim 10^{-2}$ GeV. Combining (79), (80) and (81) together, we find that our late time inflation scenario can successfully generate the required initial condition (72) if the Kähler metric is enlarged to be

$$K'' \approx 8 \times 10^{-3}. \tag{82}$$

It is not unreasonable to expect that the nonperturbative terms in the Kähler potential stabilizing $\mathrm{Re}(Z)$ provide the necessary enlargement of the Kähler metric, particularly when those terms involve a large power of $(Z + Z^*)$, e.g. $K_{np} = h(Z + Z^*)^k e^{-b\sqrt{Z+Z^*}} = \mathcal{O}(1)$ for $\mathrm{Re}(Z) \sim 32$ with $k \gtrsim 6$,

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